

Kurvenschar - Beispiel

Extrema

Kandidaten finden $f'(x) = 0$

$$f'(x) = x^2 + 2kx + 1$$

$$x_{0,1} = -k \pm \sqrt{k^2 - 1}$$

$$|k| > 1 \Rightarrow x_0 = -k + \sqrt{k^2 - 1}$$

$$x_1 = -k - \sqrt{k^2 - 1}$$

$$|k| = 1 \Rightarrow x_0 = -k$$

$$|k| < 1 \Rightarrow \text{keine NST}$$

Kandidaten prüfen

$$f'(x) = 2x + 2k$$

$$\begin{aligned} |k| > 1 \Rightarrow f''(-k + \sqrt{k^2 - 1}) &= 2(-k + \sqrt{k^2 - 1}) + 2k \\ &= -2k + 2\sqrt{k^2 - 1} + 2k \\ &= 2\sqrt{k^2 - 1} > 0 \end{aligned}$$

\Rightarrow Minimum in $x_0 = -k + \sqrt{k^2 - 1}$

$$\begin{aligned} f''(-k - \sqrt{k^2 - 1}) &= 2(-k - \sqrt{k^2 - 1}) + 2k \\ &= -2k - 2\sqrt{k^2 - 1} + 2k \\ &= -2\sqrt{k^2 - 1} < 0 \end{aligned}$$

\Rightarrow Maximum in $x_1 = -k - \sqrt{k^2 - 1}$

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$$\begin{aligned} |h| = 1 &\Rightarrow f_1''(-1) = 2(-1) + 2 \cdot 1 \\ &= -2 + 2 \\ &= 0 \Rightarrow \text{keine Aussage} \end{aligned}$$



VZW-Kriterium $x^2 + 2hx + 1$

$$\begin{aligned} f_1'(-2) &= (-2)^2 + 2(-2) + 1 \\ &= 4 - 4 + 1 \end{aligned}$$

$$\begin{aligned} f_1'(0) &= 1 + \\ &= 1 + \end{aligned} \left. \vphantom{\begin{aligned} f_1'(0) &= 1 + \\ &= 1 + \end{aligned}} \right\} \Rightarrow \text{Sattel-/Terrassenpunkt} \\ &\text{in } x_0 = -h$$

$$f_1'(-2) = (-2)^2 - 2 \cdot (-2) + 1$$

$$\begin{aligned} &= 9 + \\ f_1'(0) &= 1 + \end{aligned} \left. \vphantom{\begin{aligned} &= 9 + \\ f_1'(0) &= 1 + \end{aligned}} \right\} \Rightarrow \text{Sattel-/Terrassenpunkt} \\ &\text{in } x_0 = -h$$