Kurvenschar - Beispiel

Extrema

Kandidaten finden $f'(x) = 0$

$$f'(x) = x^2 + 2k x + 1$$

$$x_{0,1} = -k \pm \sqrt{k^2 - 1}$$

$|k| > 1 \Rightarrow$ $x_0 = -k + \sqrt{k^2 - 1}$

$x_1 = -k - \sqrt{k^2 - 1}$

$|k| = 1 \Rightarrow$ $x_0 = -k$

$|k| < 1 \Rightarrow$ keine NST

Kandidaten prüfen

$$f'(x) = 2x + 2k$$

$|k| > 1 \Rightarrow$

$$f''(-k + \sqrt{k^2-1}) = 2\,(-k + \sqrt{k^2-1}) + 2k$$
$$= -2k + 2\sqrt{k^2-1} + 2k$$
$$= 2\sqrt{k^2-1} > 0$$

$\Rightarrow$ Minimum in $x_0 = -k + \sqrt{k^2-1}$

$$f''(-k - \sqrt{k^2-1}) = 2\,(-k - \sqrt{k^2-1}) + 2k$$
$$= \cancel{-2k} - 2\sqrt{k^2-1} \cancel{+2k}$$
$$= -2\sqrt{k^2-1} < 0$$

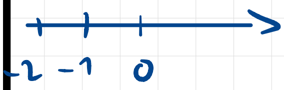
$\Rightarrow$ Maximum in $x_1 = -k - \sqrt{k^2-1}$

Kurvenschar - Beispiel

$\boxed{|k| = 1} \Rightarrow f_1''(-1) = 2(-1) + 2 \cdot 1$

$= -2 + 2$

$= 0 \Rightarrow$ keine Aussage



VZW-Kriterium $\quad x^2 + 2kx + 1$

$f_1'(-2) = (-2)^2 + 2(-2) + 1$

$= 4 - 4 + 1$

$= 1 \quad +$

$f_1'(0) = 1 \quad +$ $\Big\}$ $\Rightarrow$ Sattel-/Terassenpunkt in $x_0 = -k$

$f_{-1}'(-2) = (-2)^2 - 2 \cdot (-2) + 1$

$= 9 \quad +$

$f_{-1}(0) = 1 \quad +$ $\Big\}$ $\Rightarrow$ Sattel-/Terassenpunkt in $x_0 = -k$