4. Die Ableitungsfunktion

1. a) b) c)

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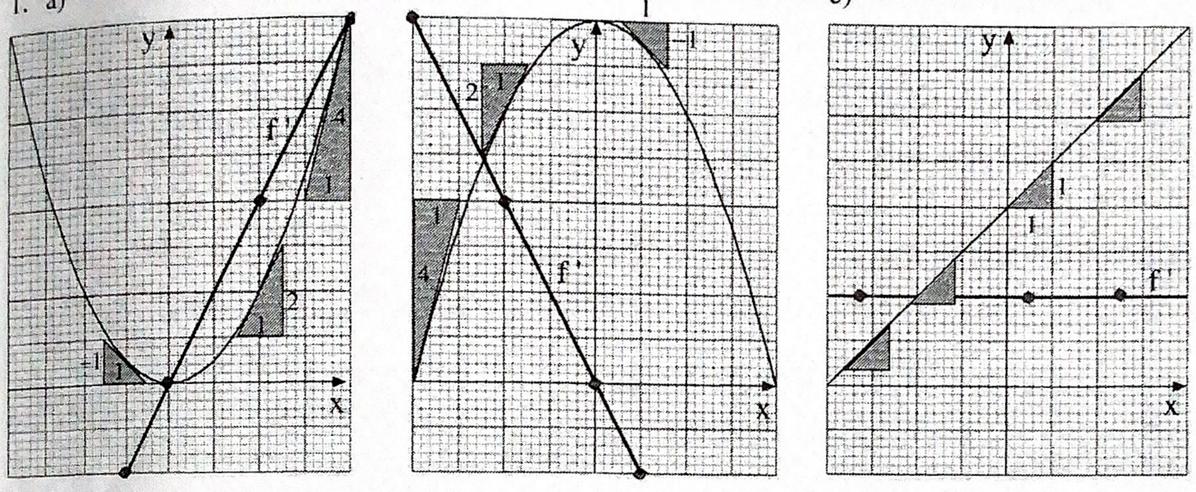
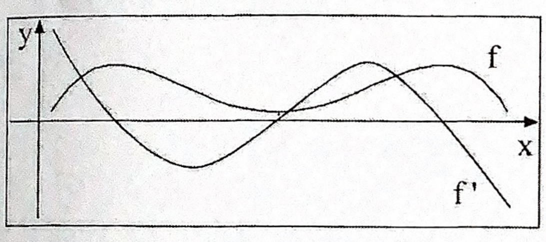


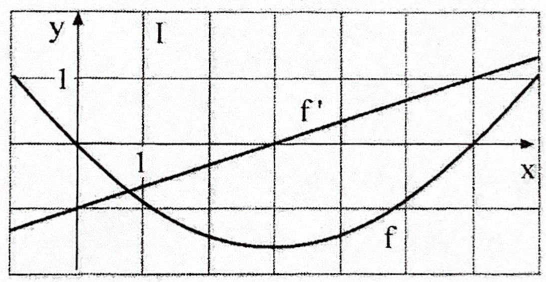
2.

95



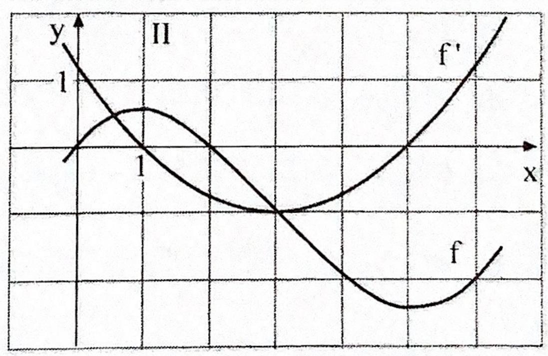
3. I a) f ist fallend für $x < 3$
und steigend für $x > 3$.
b) Der Tiefpunkt liegt bei $x = 3$.

c)



II a) f ist steigend für $x < 1$
f ist fallend für $1 < x < 5$
f ist steigend für $x > 5$
b) Der Hochpunkt liegt bei $x = 1$.
Der Tiefpunkt liegt bei $x = 5$.

c)



4. a) Nach 5 Minuten Aufstieg sind 1000m Höhe erreicht, dann geht es wieder abwärts. Nach weiteren 5 Minuten weiche Landung.
Flugdauer: 10 min
Gipfelhöhe: 1000 m

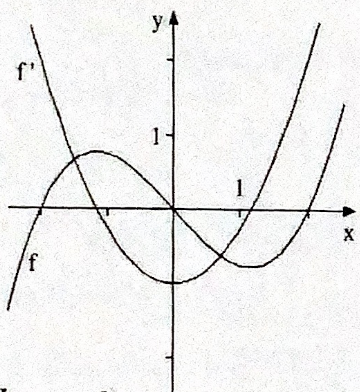
101 7. a) $f'(x) = -\frac{5}{x^6}$ b) $f'(x) = -\frac{12}{x^5}$ c) $f'(x) = \frac{2}{x^7}$

d) $f'(x) = 1 + \frac{2}{x^3} - \frac{3}{x^4}$

8. a) $f'(x) = -\frac{4}{x^3} = 0{,}5$, $x = -2$

b) $f'(x) = -\frac{4}{x^2} = -\frac{1}{9}$, $x = \pm 6$ c) $f'(x) = -\frac{12}{x^4} = -\frac{3}{4}$, $x = \pm 2$

103 9.



10. I $\rightarrow$ C II $\rightarrow$ D III $\rightarrow$ A IV $\rightarrow$ B

11. a) $f'(x) = \lim\limits_{h \to 0} \frac{\frac{1}{2}(x+h)^2 - \frac{1}{2}x^2}{h} = \lim\limits_{h \to 0} \frac{hx + \frac{1}{2}h^2}{h} = \lim\limits_{h \to 0}(x + \frac{1}{2}h) = x$

b) $f'(x) = \lim\limits_{h \to 0} \frac{2(x+h)-1-(2x-1)}{h} = \lim\limits_{h \to 0} \frac{2h}{h} = 2$

c) $f'(x) = \lim\limits_{h \to 0} \frac{(x+h)-(x+h)^2-(x-x^2)}{h} = \lim\limits_{h \to 0} \frac{h-2hx-h^2}{h} = 1 - 2x$

12. a) $f'(x) = x^3 - 4x$ b) $f'(x) = -6x$

c) $f(x) = 3x^2 - 12x + 12 + x$, $f'(x) = 6x - 11$ d) $f'(x) = 3ax^2 + 2bx + c$

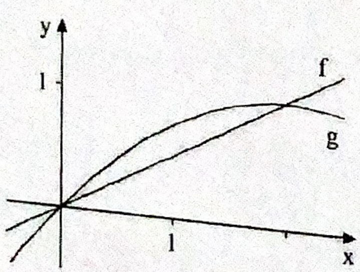
e) $f'(x) = \frac{1}{\sqrt{x}}$ f) $f'(x) = \frac{-4}{x^2}$

13. a) $f'(x) = x$, $f'(2) = 2$ b) $f'(x) = -2$, $f'(3) = -2$ c) $f'(x) = 4x - 2$, $f'(0) = -2$

d) $f'(x) = \frac{1}{2\sqrt{x}}$, $f'(4) = \frac{1}{4}$ e) $f(x) = 2x + 4\sqrt{x} + 2$, $f'(x) = 2 + \frac{2}{\sqrt{x}}$, $f'(1) = 4$

f) $f'(x) = 1 - \frac{1}{x^2}$, $f'(1) = 0$

14. a)



b) $f'(x) = 0{,}5$, $f'((1) = 0{,}5$
$g'(x) = -0{,}5x + 1$, $g'(1) = 0{,}5$

c) $m_f = 0{,}5$, $m_g = \frac{1-0}{2-0} = 0{,}5$

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15\. a) $f'(x) = x^3 - 6 = 2$, $x = 2$ b) $f'(x) = -0{,}5x^2 + 2x = -2{,}5$, $x = 5$ und $x = -1$

c) $f'(x) = -\frac{2}{x^2} - 1 = -3$, $x = \pm 1$ d) $f'(x) = \frac{3}{2\sqrt{x}} = 3$, $x = \frac{1}{4}$

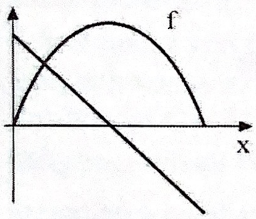
16\. a) $f'(x) = g'(x)$: $x = 2$ b) $x = \pm 1$ c) $x = \pm 3$

d) $x = 4$

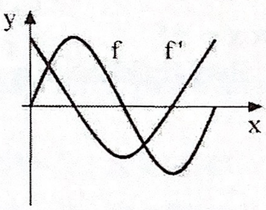
17\. a) $(-4x^2)' = -8x$, $5' = 0$ b) $(x^{-1})' = -x^{-2}$ c) $(c^3)' = 0$

d) $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$, $(\frac{1}{x})' = -\frac{1}{x^2}$

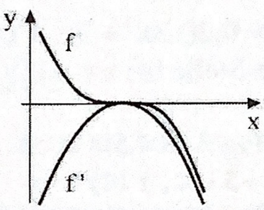
18\. a)



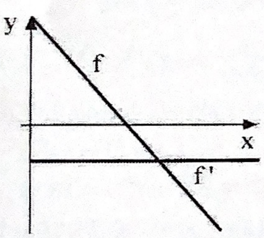
b)



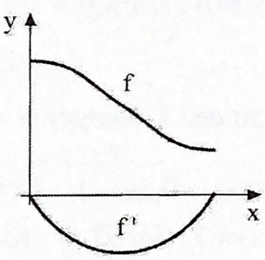
c)



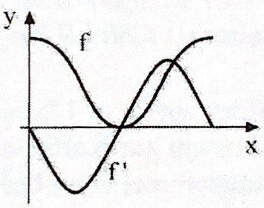
d)



e)



f)



19\. a) $f'(x) = x$, $x = 1$ b) $f'(x) = \frac{1}{2\sqrt{x}} = \tan 30° = \frac{1}{\sqrt{3}}$, $x = \frac{3}{4}$

c) $f'(x) = \frac{3}{\sqrt{x}} = \tan 60° = \sqrt{3}$, $x = 3$ d) $f'(x) = -\frac{3}{x^2} = -1$, $x = \pm\sqrt{3} \approx \pm 1{,}73$

Knobelaufgabe
Vor einer Stunde hatte James x Centmünzen und y Euromünzen.
Danach hatte er noch y Centmünzen und x Euromünzen mit dem halben Wert.
$xC + yE = 2(yC + xE)$, $(2y - x)C + (2x - y)E = 0$, mit $1E = 100C$ folgt

$(2y - x)C + (2x - y)100C = 0$, $199x - 98y = 0$, d.h. $\frac{x}{y} = \frac{98}{199}$

199,98 Euro ist doppelt so viel wert wie 98 Euro und 199 Cent nämlich 99,99 Euro.